

Evaporation of the gluon condensate: a model for pure gauge $SU(3)_c$ phase transition

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Abstract

We interpret lattice data for the equation of state of pure gauge $SU(3)_c$ by an evaporation model. At low temperatures gluons are frozen inside the gluon condensate, whose dynamics is described in terms of a dilaton lagrangian. Above the critical temperature quasi-free gluons evaporate from the condensate: a first order transition is obtained by minimizing the thermodynamical potential of the system. Within the model it is possible to reproduce lattice QCD results at finite temperature for thermodynamical quantities such as pressure and energy. The gluonic longitudinal mass can also be evaluated; it vanishes below the critical temperature, where it shows a discontinuity. At very large temperatures we recover the perturbative scenario and gluons are the only asymptotic degrees of freedom.

In recent years, precise data have become available concerning QCD thermodynamics at high temperature via numerical simulation on a lattice (for a recent review, see [1]). Data exist now both for the pure gauge sector and for complete QCD at zero chemical potential; the latter has been explored both in the limit of infinite quark masses and in the chiral limit. Moreover, calculations for finite values of quark masses and for a non-vanishing chemical potential are now appearing; the availability of such a large number of new lattice data surely represents an important opportunity to test the effectiveness of models in reproducing the finite temperature phase transition.

In this paper, we will concentrate on the simplest case, i.e., the pure gauge sector. The main known characteristics of $SU(3)_c$ at finite temperature are the following. A first order transition takes place at a temperature $T_c = (271 \pm 2)$ MeV and at $T \sim \text{few } T_c$ the asymptotic limit of a Stefan-Boltzmann gas is not yet reached. A small but not negligible value for pressure, entropy and energy at T just below T_c has been computed and the size of the discontinuity of the energy at T_c , representing the latent heat of the transition, has also been estimated.

In our work, we interpret lattice data by assuming a theoretical scenario similar to the one suggested by Simonov [2]. In this approach, at $T \leq T_c$ the dynamics of the gluon condensate is dominant, while at $T > T_c$ the condensate evaporates in the form of quasi-free gluons.

Several models have been used to describe lattice data (for a review, see [3]). Early attempts were based on the MIT bag model [4], but more sophisticated approaches became necessary when more precise lattice data started appearing: for instance, the results for the energy density, the pressure and the entropy of a pure gluon system at $T > T_c$ can be well reproduced by quasi-particle models in which gluons acquire an effective temperature-dependent mass [5, 6, 7]. More recently, it has been pointed out that the number of effective degrees of freedom (i.e. the gluon degeneracy of the system) can itself be temperature dependent [8, 9].

An interesting idea, similar to the one we will use in this work, is implemented in the so-called “cut-off” model [10, 11, 12], in which gluons having momenta smaller than a fixed value K are bound inside non-perturbative structures and therefore do not directly contribute to the thermodynamics of the system.

In the models discussed so far, two important limitations are present:

firstly, the critical temperature T_c plays the role of a parameter and cannot be computed within the model; then, and more important, the transition itself is parametrized and not obtained dynamically, for instance through the minimization of the energy of the system. In the present work we will try to overcome such limitations: in our approach it is possible to describe the thermodynamical behavior of the system and obtain a first order transition via the minimization of the thermodynamical potential; the value of the critical temperature at which the transition takes place can also be estimated ¹.

In our model, we consider three different contributions to the thermodynamical potential: the first component comes from the gluon condensate, whose dynamics is expressed in terms of a dilaton lagrangian [14, 15, 16, 17]; then, gluons are introduced in a way similar to the one used in the cut-off model [10, 11, 12]. At variance with the previous versions of that model, in our case the cut-off itself is not a parameter, but rather a function of the expectation value of the dilaton field, i.e. of the gluon condensate. It seems rather natural to assume that, when the gluon condensate is large, namely at low temperature, many gluons are frozen inside this non-perturbative structure: as a consequence, the infrared cut-off is large. On the contrary, when the value of the gluon condensate is reduced, gluons having large enough momenta may “evaporate”, and behave as almost-free particles.

Finally, the last contribution to the thermodynamical potential is due to the perturbative gluon-gluon interaction, which is present even at large temperature, i.e. for $T \gg T_c$. In the following, we will discuss in sequence these three contributions.

1 Gluon-condensate dynamics

The idea of a gluon condensate has been introduced many years ago [18, 19]. In our approach, we will not try to obtain a mechanism for gluon condensation, namely a model for the QCD vacuum, but we will instead describe the dynamics of the gluon condensate by introducing an effective degree of freedom, i.e. the dilaton field.

In fact, there is a deep connection between the gluon condensation phe-

¹A similar approach has been discussed in Ref. [13], where the effective degrees of freedom are the dilaton field and constituent gluons which become massive via an interaction with the gluon condensate. However, the results obtained in Ref. [13] are not completely satisfactory at temperatures just above the critical one.

nomenon and the violation of the scale invariance, which, in QCD at the first loop, is quantified by:

$$\langle \partial_\mu j_{QCD}^\mu \rangle = -\frac{11N_c}{96\pi^2} \langle g^2 G^2 \rangle, \quad (1)$$

where j_{QCD}^μ is the dilatation current in QCD and G^2 is the gluon field strength. In order to reproduce the QCD scale anomaly and to satisfy low-energy theorems [20], a dilaton field has been introduced [16, 17] whose lagrangian reads

$$L_{\text{dil}} = \frac{1}{2}(\partial_\mu \sigma)^2 - V(\sigma), \quad (2)$$

where

$$V(\sigma) = \frac{B}{4} \left[\sigma_0^4 - \sigma^4 + 4 \sigma^4 \ln \left(\frac{\sigma}{\sigma_0} \right) \right]. \quad (3)$$

The violation of the scale invariance is given by:

$$\partial_\mu j_{\text{dil}}^\mu = 4V - \frac{\partial V}{\partial \sigma} \sigma = -B\sigma^4 \quad (4)$$

and it must satisfy the equality

$$\langle \partial_\mu j_{\text{dil}}^\mu \rangle = \langle \partial_\mu j_{QCD}^\mu \rangle. \quad (5)$$

The potential $V(\sigma)$ has a minimum at $\sigma = \sigma_0$, where $V(\sigma_0) = 0$; the small oscillations around the minimum correspond to the excitations of a scalar glueball and they can be parametrized as follows:

$$\begin{aligned} V(\sigma) &\simeq 2B\sigma_0^2 (\sigma - \sigma_0)^2 + O[(\sigma - \sigma_0)^3] \\ &\equiv \frac{1}{2} M_g^2 (\sigma - \sigma_0)^2 + O[(\sigma - \sigma_0)^3], \end{aligned} \quad (6)$$

where the glueball mass M_g is

$$M_g = 2 \sigma_0 \sqrt{B}. \quad (7)$$

The dilaton potential contains two parameters, σ_0 and B , which can be related through eqs. (1, 4, 5, 7) to the value of the gluon condensate in the vacuum and to the mass of the scalar glueball. Concerning the estimate of the gluon condensate (for a review, see [21]), Ref. [22] indicates the range

$$0.12 \text{ GeV}^4 \leq \langle (gG)^2 \rangle \leq 0.83 \text{ GeV}^4, \quad (8)$$

while both Refs. [23] and [24] indicate a value $\langle (gG)^2 \rangle \sim 0.5 \text{ GeV}^4$ associated, in Ref. [24], to an error of about 50%.

The mass of the scalar glueball has been recently estimated in lattice QCD, obtaining a mass $M_g = (1730 \pm 30 \pm 80) \text{ MeV}$ [25].

The two parameters σ_0 and B are therefore constrained into a relatively narrow window, the uncertainty being mainly due to the error bar in the estimate of the gluon condensate.

In the following, we will study the thermodynamical potential associated with the dilaton lagrangian, at the mean field level, by using the standard techniques of finite temperature field theory [26, 27].

The thermodynamics of the dilaton field at finite temperature has already been discussed in the literature, for instance in Refs. [28] and [29]. In Ref. [29] an attempt to go beyond the mean-field approximation was made, and in Ref. [30] the gluon condensate was studied with renormalization group flow equations. We will compare later our results with the ones of Ref. [30]. Due to the difficulties associated with the quantization of non-polynomial field theories (see e.g. [31]) we prefer to stick to the mean-field approximation; moreover, it is not possible to apply the renormalization group techniques to our complete model which incorporates also gluons evaporating from the condensate.

In the mean field approximation the thermodynamical potential reads:

$$\Omega_{\text{dil}}(\sigma, T) = V(\sigma) - P_{\text{dil}}(\sigma, T), \quad (9)$$

where P_{dil} , the pressure of the dilaton field, reads²

$$P_{\text{dil}}(\sigma, T) = -T \int \frac{d^3p}{(2\pi)^3} \ln \left[1 - e^{-\omega/T} \right], \quad (10)$$

and

$$\omega = \sqrt{\mathbf{p}^2 + [m(\sigma)]^2}. \quad (11)$$

The σ -dependent mass $m(\sigma)$ is defined as

$$m^2(\sigma) = \frac{\partial^2 V}{\partial \sigma^2} \quad (12)$$

²At very large temperatures the contribution of the dilaton field to the thermodynamical quantities should vanish. To this purpose, in Sec. 2.1.1 an ultraviolet cut-off will be introduced in eq. (10).

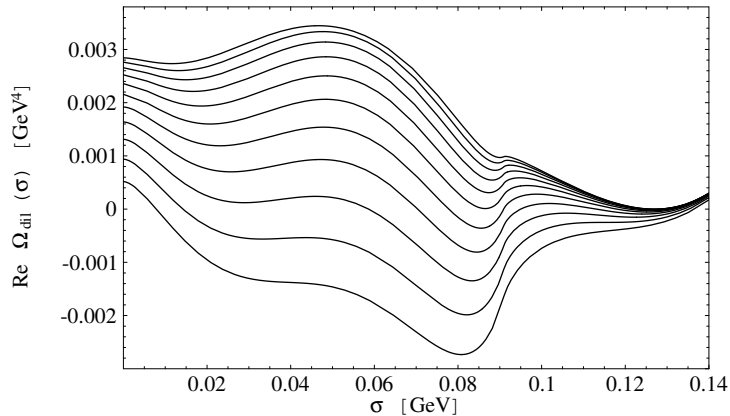


Figure 1: Real part of dilaton thermodynamical potential for various temperatures. Temperature increases from upper to lower curves, ranging from 0.2 GeV to 0.4 GeV in steps of 0.02 GeV.

and it equals M_g^2 for $\sigma = \sigma_0$. The mass squared is negative for $\sigma < e^{-1/3}\sigma_0$ and therefore the thermodynamical potential gets an imaginary part for small values of σ . There is in the literature a broad discussion about the physical interpretation of a complex potential and how to deal with it (see e.g. [32]). We will follow Ref. [33], where the imaginary part is interpreted as a signal of the instability of the system. Our recipe for dealing with a complex thermodynamical potential is therefore all simply to minimize its *real* part. Actually the problem of interpreting the imaginary part of the potential, although conceptually important, turns out to be not so important from a practical viewpoint, since after the introduction of gluons (what will be done in the next Section) the dilaton field will vanish for $T > T_c$ and we will not have to deal with the instability region.

In Fig. 1 we present $\text{Re } \Omega_{\text{dil}}(\sigma, T)$, as a function of σ , for various temperatures. As one can clearly see, when the temperature increases the real part of the thermodynamical potential develops a new minimum for $\sigma < \sigma_0$. At the critical value T_c , the new minimum becomes the absolute one and a first order transition takes place. We must stress again that the transition of the pure dilaton field appears to be first order due to the approximations we have used. The order of the transition could be established in a consistent way only using a non-perturbative approach, e.g. studying the dilaton potential

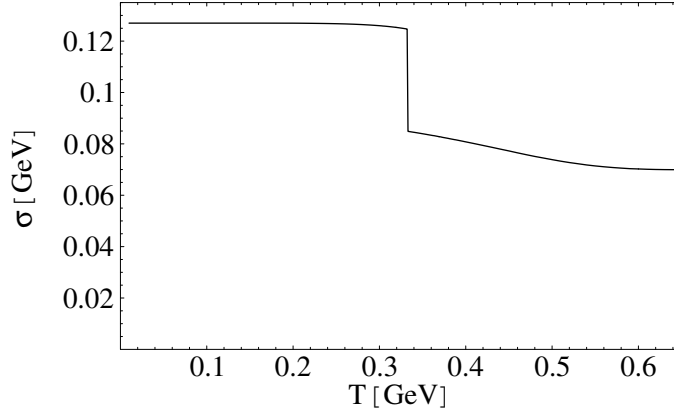


Figure 2: Dilaton field as a function of the temperature in the pure dilatonic model.

on the lattice. One should also consider the results of Ref. [30], where no hint of a first order transition was found up to temperatures of the order of 200 MeV where their prediction reaches its limit of validity. On the other hand, many calculations of the behavior of the dilaton field at finite temperature do indicate a transition at a temperature similar to the one we get in our approach [28, 29, 32]. It is also interesting to notice that, since in the dilaton lagrangian the only dimensional parameter is σ_0 , the critical temperature as a function of B and σ_0 must be of the form:

$$T_c^{\text{dil}} = f(B) \sigma_0, \quad (13)$$

where $f(B)$ is a function to be determined numerically by minimizing $\text{Re } \Omega_{\text{dil}}$.

In Fig.2, we present the expectation value of the dilaton field as a function of the temperature: for $T < T_c$ there is a very small reduction of σ from its zero-temperature value σ_0 . This shift of the expectation value of the dilaton field corresponds to thermal excitations of the glueball. At the critical temperature T_c^{dil} , the dilaton field is discontinuous. Note that for $T > T_c^{\text{dil}}$ the dilaton field does not vanish. We will see later on that the contribution of quasi-free gluons shifts the value of σ to zero in the deconfined phase. Figs. 1 and 2 have been obtained using $B = 46.4$ and $\sigma_0 = 0.127$ GeV, which correspond to $\langle (gG)^2 \rangle = 0.35$ GeV⁴ and $M_g = 1.73$ GeV. The critical temperature, in absence of gluons is $T_c = T_c^{\text{dil}} = 0.3$ GeV: the extra pressure of the gluons in the deconfined phase reduces this value, bringing it close to the one indicated by lattice calculations.

2 The evaporation model

2.1 Quasi-free gluons

In the previous section we have discussed the behavior of the dilaton field at finite temperature. It is clear that its excitations (i.e. the glueballs) cannot represent the relevant degrees of freedom at large temperature, where quasi-free gluons should give the dominant contribution to the thermodynamical observables. On the other hand, quasi-free gluons should be suppressed below T_c , which, in our model, is the temperature at which the dilaton field is discontinuous. A simple way to suppress the quasi-free gluons in the confinement region is by assuming that they are frozen inside the gluon condensate: when the value of the gluon condensate is large, i.e. below T_c , most of the gluons are frozen while, above T_c , the condensate evaporates and gluons become quasi-free particles. Technically, this idea can be implemented by introducing an infrared cut-off K in the gluon distribution function, so that only gluons having a momentum larger than K contribute to the thermodynamics of the system:

$$P_{\text{q-free}}(\sigma, T) = -2(N_c^2 - 1)T \int \frac{d^3k}{(2\pi)^3} \ln [1 - e^{-k/T}] \Theta(k - K(\sigma)) \quad (14)$$

In our model we assume that the cut-off K is a function of the gluon condensate, i.e. of the expectation value of the dilaton field. The “evaporation” model has been already discussed in Refs. [10, 11, 12], but there the cut-off was assumed to be a fixed parameter. We use for the cut-off the form

$$K(\sigma) = \frac{A}{\left(\frac{\sigma_0 - \sigma}{\sigma_0}\right)^\alpha}, \quad (15)$$

so that if $\sigma \rightarrow \sigma_0$, then $K \rightarrow \infty$, while if $\sigma \rightarrow 0$ then $K \rightarrow A$. It can be interesting to notice that we can not reproduce the lattice data satisfactorily if we use a cut-off which vanishes for $\sigma \ll \sigma_0$. This result seems to indicate that, even at large temperatures, at which $\sigma \sim 0$, we gluons are still suppressed. The value of the parameter A is of the order of 1 GeV and it is therefore natural to interpret this parameter as the one regulating the transition from the perturbative behavior of the gluon propagator to the non-perturbative one.

2.1.1 High temperature degrees of freedom

At very large temperature the perturbative degrees of freedom should be recovered and therefore the dilaton field cannot appear as an effective degree of freedom for $T \gg T_c$. On the other hand, at temperatures just above T_c scalar gluon-gluon correlations described in terms of the dilaton field could still be relevant. We will see in Sec. 3 that, above T_c , $\sigma \sim 0$; as a consequence the dilaton mass vanishes and scalar gluon correlations can exist but with a vanishing mass gap [28].

The idea of describing correlations between the asymptotic degrees of freedom in terms of effective fields is adopted in many physical problems. For instance σ -models have been used in studying the chiral phase transition [34]³. In that case, the chiral fields describe quark-antiquark correlations in an effective non-perturbative way below and above T_c . In the scheme we are discussing it is possible, at least in principle, to provide a structure for the dilaton field in terms of gluonic degrees of freedom [2]. For simplicity we mimic the dynamics of gluons inside the dilaton by introducing an ultraviolet cut-off in the dilaton pressure. Since above T_c gluons having a momentum larger than K are quasi-free, and since at least two gluons are needed to produce a scalar correlation, we assume that correlations having a momentum larger than $2K$ are suppressed and do not contribute to the thermodynamical quantities.

We modify therefore eq. (10) by introducing an ultraviolet cut-off equal to $2K$. In this way, at very large temperatures, the dilaton field does not contribute, as shown in Fig. 5. On the other hand, the effect of this cut-off is almost negligible at $T \sim$ a few T_c .

2.2 Residual perturbative interaction

Lattice data clearly indicate that even at a temperature larger than $4T_c$ the Stefan-Boltzmann limit is not yet reached and the data for pressure, energy and entropy lie below the free-gas limit. As we will see, the introduction of an infrared cut-off is not sufficient to explain both the data near T_c and those at large temperatures. It is therefore necessary to introduce perturbative corrections, which have a relevant role for $T > T_c$. These corrections are well

³Similarly, in the study of hadronic structure, pions can be introduced as effective degrees of freedom, whose substructure can later be resolved in terms of the quark distribution function of the pion (see for instance [35, 36]).

established and a huge amount of work has been devoted to their estimate (for a recent review see [37]). In the present work we are not interested in a detailed comparison with the data at $T \gg T_c$, but we mainly aim at describing the data near the critical temperature. We have therefore adopted the simplest prescription, which consists in taking into account only the first order $O(g^2)$ corrections. These corrections have been computed in Ref.[38], but in the present case additional Θ -functions occur which restrict the phase space for the interacting gluons (see [3]). The $O(g^2)$ contribution to the pressure reads:

$$\begin{aligned}
P_{\text{int}}(\sigma, T) = g^2 N_c (N_c^2 - 1) & \left\{ -3 \left(\int \frac{d^3 k}{(2\pi)^3} \frac{1}{k} N_B \left(\frac{k}{T} \right) \Theta[k - K(\sigma)] \right)^2 \right. \\
& + \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \frac{1}{k_1 k_2} N_B \left(\frac{k_1}{T} \right) N_B \left(\frac{k_2}{T} \right) \Theta[k_1 - K(\sigma)] \Theta[k_2 - K(\sigma)] \\
& \left. \times \left(\frac{9}{4} \Theta[|\mathbf{k}_1 + \mathbf{k}_2| - K(\sigma)] - \frac{1}{4} \Theta[|\mathbf{k}_1 - \mathbf{k}_2| - K(\sigma)] \right) \right\} , \quad (16)
\end{aligned}$$

where $N_B(x) = (e^x - 1)^{-1}$ is the Bose-Einstein distribution. Here g^2 is the temperature-dependent running coupling-constant

$$g^2(T) = \frac{48\pi^2}{11N_c \ln[(T^2 + S^2)/\Lambda^2]} , \quad (17)$$

where we have introduced a regulator S , whose appearance can be related to the existence of a minimal momentum K for the propagating gluons [3]⁴. As we shall see, the perturbative corrections play a relatively minor role in our model and more sophisticated choices of the running coupling would hardly affect the results. The typical value for S is $S \sim \text{GeV}$.

⁴In principle, g^2 can also depend on the value of the gluon condensate [39]. This possibility will be explored in a future work [40].

3 Results

The parameters in our model are the following:

- dilaton lagrangian: B, σ_0
- quasi-free gluons: A, α
- running coupling-constant: Λ, S .

Concerning B and σ_0 , as discussed in Sec. 1 their value is bounded by the “experimental” value of the gluon condensate and by the lattice result for the mass of the scalar glueball M_g . In the following we will present results obtained using $B = 46.4$ and $\sigma_0 = 0.127$ GeV, which correspond to $\langle (gG)^2 \rangle = 0.35$ GeV⁴ and to $M_g = 1.73$ GeV, both near the center values indicated for these quantities. Concerning the quasi-free gluons, we use $A = 1.015$ GeV and $\alpha = 0.5$. Finally, the parameters for the running coupling-constant are not too strictly constrained in our calculation, since the perturbative interaction turns out to play a minor role in our model. We adopted $S = 7.15$ GeV and $\Lambda = T_c$, where T_c is the value of the critical temperature computed in the model. These parameter values are also consistent with the more general form for g^2 introduced in [40] (see note 4). Notice that the value of the critical temperature is not modified by the perturbative corrections and can therefore be computed before the latter are taken into account.

3.1 Thermodynamical quantities

The results we present are obtained by minimizing the real part of the total thermodynamical potential $\Omega_{\text{tot}}(\sigma, T)$, as a function of σ , for a given temperature T , where:

$$\Omega_{\text{tot}}(\sigma, T) = \Omega_{\text{dil}}(\sigma, T) - P_{\text{q-free}}(\sigma, T) - P_{\text{int}}(\sigma, T) . \quad (18)$$

In Fig. 3 we present the real part of the thermodynamical potential. Due to the extra pressure of quasi-free gluons, a new minimum develops for $\sigma = 0$. In this way we avoid entering the region of instability of the dilaton field, where imaginary parts develop, since the expectation value of the field jumps from a value $\sigma \sim \sigma_0$ to $\sigma = 0$. The value of the critical temperature, which corresponds to the discontinuity of the dilaton field, is $T_c = 0.27$ GeV. The reduction of the value of the critical temperature, due to the introduction

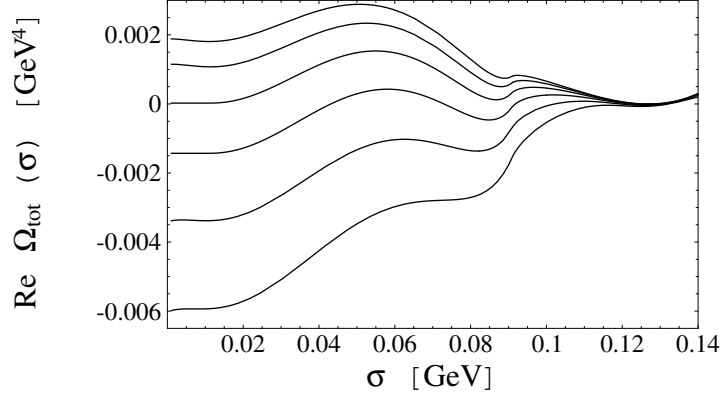


Figure 3: Real part of total thermodynamical potential for various temperatures. Temperature increases from upper to lower curves, ranging from 0.23 GeV to 0.33 GeV in steps of 0.02 GeV.

of gluons in the deconfined phase, is rather independent of the precise value of the cut-off parameters and it turns out to be always of the order of 10%. The critical temperature T_c^{dil} is therefore a rather good approximation to the value of T_c as computed from the complete model.

In Fig. 4 we compare our result for the scaled pressure with the lattice data [1]. The pressure is obviously connected to the thermodynamical potential by the relation $P = -\Omega_{\text{tot}}$. We also show the pressure obtained minimizing the thermodynamical potential neglecting the interaction term P_{int} . It is clear that the interaction modifies significantly the pressure only for temperatures of the order of $2T_c$, or larger. The critical temperature, as well as the shape of the pressure near T_c , are independent of the interaction contribution. As already stated, we are not particularly interested in reproducing in a very accurate way the data at large temperatures, introducing perturbative corrections in a sophisticated way, our main aim being to describe the phase transition.

In Fig. 5 we present the decomposition of the total pressure into its various contributions. As it can be seen, the dilaton contribution and the quasi-free-gluons one are both discontinuous at the critical temperature, but their discontinuities cancel so that the total pressure is continuous. In Ref.[2], such a behavior for the gluon condensate and for the quasi-free gluons was anticipated. From the figure it is also clear that the perturbative correction

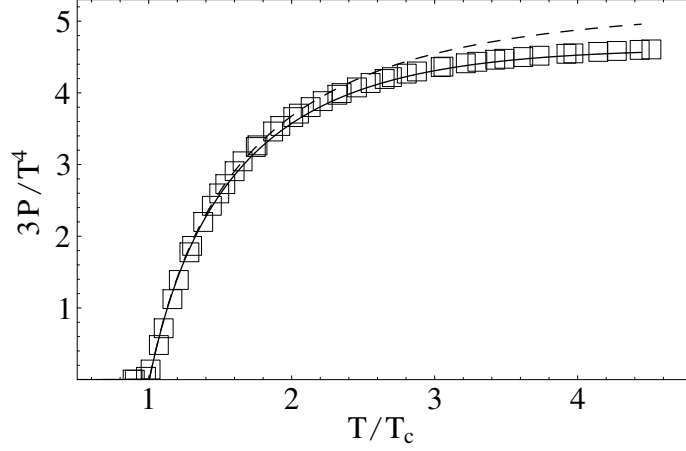


Figure 4: Scaled pressure in our model compared with lattice data. The solid line is obtained minimizing the total thermodynamical potential, while the dashed line is obtained neglecting P_{int} .

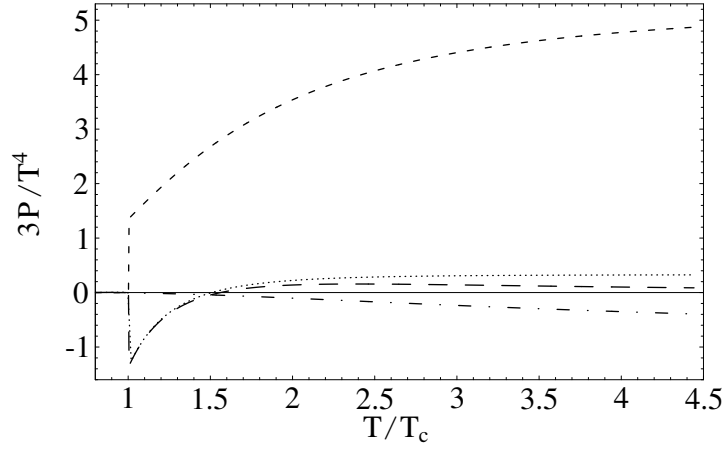


Figure 5: Various contributions to the total pressure. The long-dashed line corresponds to $P_{\text{dil}} = -\Omega_{\text{dil}}$, the short-dashed line is $P_{\text{q-free}}$, the dashed-dotted line is P_{int} and the dotted line is P_{dil} but without the ultraviolet cut-off.

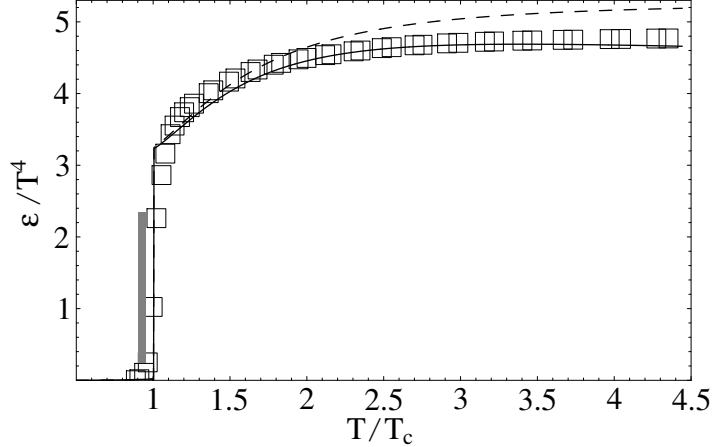


Figure 6: Scaled energy density in our model compared with lattice data. The solid line is obtained minimizing the total thermodynamical potential, while the dashed one is obtained neglecting P_{int} . The shaded rectangle indicates the latent heat as estimated in lattice calculations.

P_{int} vanishes at T_c . Let us remark again that the dilaton gives a contribution to the scaled pressure which vanishes at large temperatures, due to the presence of the ultraviolet cut-off.

In Fig. 6 we compare the energy density $\epsilon = T \frac{dP}{dT} - P$ computed in our model with lattice data⁵. We also show the lattice result for the latent heat. The main difference between our result and the lattice one is that at $T < T_c$ our energy density is considerably smaller than the one indicated by lattice calculations. This discrepancy is due to the presence, in our calculation, of the scalar glueball only, while the $J = 2$ glueball should also contribute. The mass of the latter has been estimated to be $M_{2++} = 2400 \pm 25 \pm 120$ [25]. The introduction of these new degrees of freedom would correspond, roughly, to a degeneracy factor of 6 in front of the glueball contribution [26], and would bring the computed energy density close to the lattice one at $T < T_c$. For simplicity we have not included the excitations of the tensor glueball in our calculation, but it can obviously be done in the future.

We must also notice that lattice data for the energy decrease much faster than the results of our model for temperatures just above T_c . We have found

⁵We computed also the entropy density in our model. Considerations similar to the ones done for the energy density can be done for the entropy.

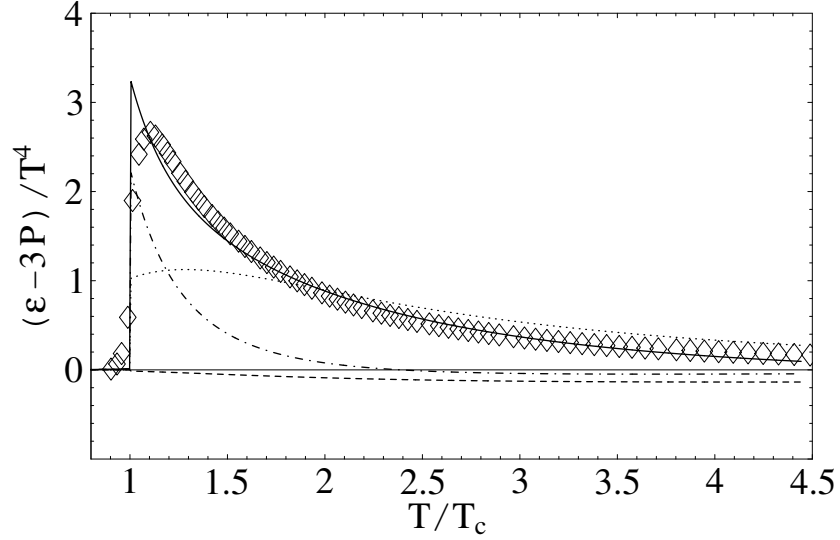


Figure 7: Interaction measure. The solid line is the total value, the dot-dashed is the dilaton contribution, the dotted is the one due to quasi-free gluons and the dashed corresponds to the perturbative interaction.

that, using other parameter sets, it is possible in principle to reduce this discrepancy. However, in those cases the dilaton field does not jump directly to zero at T_c , but it reaches a small but finite value. Due to the difficulties associated with the mean field treatment of the model for small values of σ we have not explored this possibility in detail.

Finally, in Fig. 7 we show the various contributions to the “interaction measure”, namely the (scaled) quantity indicating the distance from the Stefan-Boltzmann relation $\epsilon = 3P$. For $T \sim T_c$ the main contribution to this “interaction measure” comes from the dilaton field. The contribution due the quasi-free gluons is large at moderate temperature, but it decreases more rapidly than the contribution due to P_{int} , which is the dominant term at very high temperature.

3.2 Thermal gluon masses

In a covariant gauge, the gluon propagator can be written in the following general form

$$D_{\mu\nu} = \frac{1}{F - q^2} P_L^{\mu\nu} + \frac{1}{G - q^2} P_T^{\mu\nu} + \frac{\xi q_\mu q_\nu}{(q^2)^2} \quad (19)$$

where $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ are the longitudinal and transverse projection operators, defined as

$$\begin{aligned} P_T^{00} &= P_T^{0i} = P_T^{i0} = 0 \\ P_T^{ij} &= \delta^{ij} - q^i q^j / \mathbf{q}^2 \\ P_L^{\mu\nu} &= q^\mu q^\nu / q^2 - g^{\mu\nu} - P_T^{\mu\nu}. \end{aligned} \quad (20)$$

The gluon self-energy is given by

$$\Pi^{\mu\nu} = G P_T^{\mu\nu} + F P_L^{\mu\nu}, \quad (21)$$

G and F being scalar functions of q^0 and $|\mathbf{q}|$. The electric and magnetic masses are defined as

$$\begin{aligned} F(0, \mathbf{q} \rightarrow 0) &= -\Pi_{00}(0, \mathbf{q} \rightarrow 0) = m_{el}^2 \\ G(0, \mathbf{q} \rightarrow 0) &= \frac{1}{2} \Pi_{ii}(0, \mathbf{q} \rightarrow 0) = m_{mag}^2, \end{aligned} \quad (22)$$

where the relation between F , G and Π comes from eqs. (20) and (21).

The gluon self-energy can be evaluated perturbatively; we will concentrate on the electric mass, due to the difficulties associated with the magnetic one. Two different contributions arise for Π^{00} , a zero temperature one, which vanishes after renormalization, and a finite temperature one, which reads

$$\Pi^{00} = -12g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\omega} \frac{1}{e^{\beta\omega} - 1}. \quad (23)$$

In our model, which allows contributions to the thermodynamics of the system only from gluons having a momentum larger than K , we rewrite the above formula as:

$$\Pi^{00} = -12g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\omega} \frac{1}{e^{\beta\omega} - 1} \theta(p - K(\sigma)). \quad (24)$$

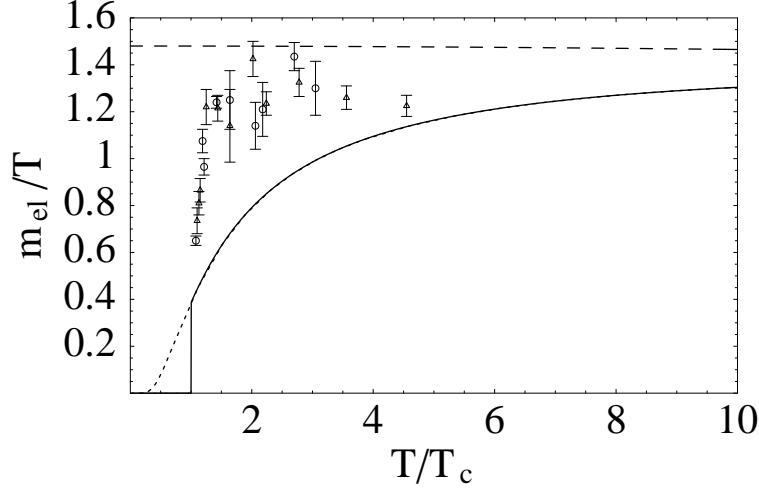


Figure 8: Gluonic electric mass in our model (solid line), and in a purely perturbative calculation without cut-off (dashed line). The data, taken from Ref. [41], correspond to half the screening mass $\mu(T)/2$ (see text). The dotted line indicates m_{el} in the metastable phase.

In Fig. 8 we show our results; the gluonic electric mass presents a discontinuity at $T = T_c$, because of the discontinuity of the σ field. We also show (dotted line) m_{el} in the supercooled, metastable phase in which σ is kept equal to $\sigma(T = T_c^+)$ for $T < T_c$ (see page. 152 of Ref. [26]). For comparison we present the electric mass in a standard perturbative calculation without the cut-off, which yields $m_{el} = gT$ (dashed line).

To compare our results with lattice QCD calculations we must recall that a screening mass $\mu(T)$ can also be introduced by considering the behavior of the potential $V(R, T)$ between gauge invariant sources at $T > T_c$. This potential can be parametrized as [41]:

$$\frac{V(R, T)}{T} = \frac{e(T)}{(RT)^d} e^{-\mu(T)R} \quad (25)$$

where d , $e(T)$ and $\mu(T)$ are determined from lattice results. In perturbative calculations, the screening mass $\mu(T)$ and the gluon electric mass m_{el} are connected by the simple relation

$$\mu = 2m_{el} . \quad (26)$$

In Fig. 8 we see that this perturbative relation is recovered for $T \gg T_c$. For temperatures just above T_c the situation is less clear, also due to the large error bars in lattice data, but the simple proportionality indicated by eq. (26) seems to be (slightly) violated.

Concerning the magnetic mass, it is well known that it vanishes in a perturbative calculation. Notice anyway that, due to the infrared cutoff which characterizes the evaporation model, higher order loops are not expected to be affected by infrared divergences, even in the absence of a magnetic mass. For the same reason, in a cut-off model like the one we have discussed here, perturbative corrections can be computed in principle up to an arbitrary order; moreover, all these contributions vanish at $T = T_c^+$.

4 Conclusions

We have presented a simple realization of an evaporation model, in which at low temperature gluons are frozen inside the non-perturbative condensate while at high temperature they escape from the condensate and behave as quasi-free particles. We have shown that it is possible within the model to reproduce the main results of lattice QCD for thermodynamical quantities such as pressure and energy. At variance with other models for these quantities, in our approach a first order transition is obtained by minimizing the thermodynamical potential and the latent heat can be estimated. It is also possible within the model to study finite temperature gluon masses. Here again our results are consistent with the indications of lattice QCD.

The main problem in the present analysis arises from the difficulties associated with the appearance of an imaginary part in the thermodynamical potential. We assumed that the imaginary part signals an instability of the system and we have therefore minimized the real part of the thermodynamical potential. This difficulty is clearly related to the mean-field approximation we have adopted. Although this technique seems sufficient to reproduce rather precisely lattice QCD results, it is clear that only more sophisticated approximations can clarify the details of the behavior of the system near the critical temperature. However, in the complete model, which includes gluons in the deconfined phase, the expectation value of the dilaton field is such that the system does not enter the unstable region at any temperature.

This calculation can be extended in several directions. Firstly, it will be important and, probably, relatively easy to study the behavior of the

phase transition as a function of the color number N_c . Lattice calculations discussing the dependence on N_c of the critical temperature, of the glue-ball masses and of the gluon condensate appeared recently (see for instance Refs. [42, 43]), allowing comparisons with our model. As pointed out in [2], in this model the value of the critical temperature is N_c independent in the large N_c limit.

Another, more important extension of the present work will be the inclusion of quarks. This can be done by dressing the quark propagator via the Schwinger-Dyson equation and by using a quark-gluon coupling which depends on the value of the gluon condensate. Work along these lines is now in progress [40].

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